Rossmoyne Senior High School

Semester Two Examination, 2018

Question/Answer booklet

MATHEMATICS METHODS UNITS 1 AND 2

Section Two: Calculator-assumed

SOLUTIONS

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

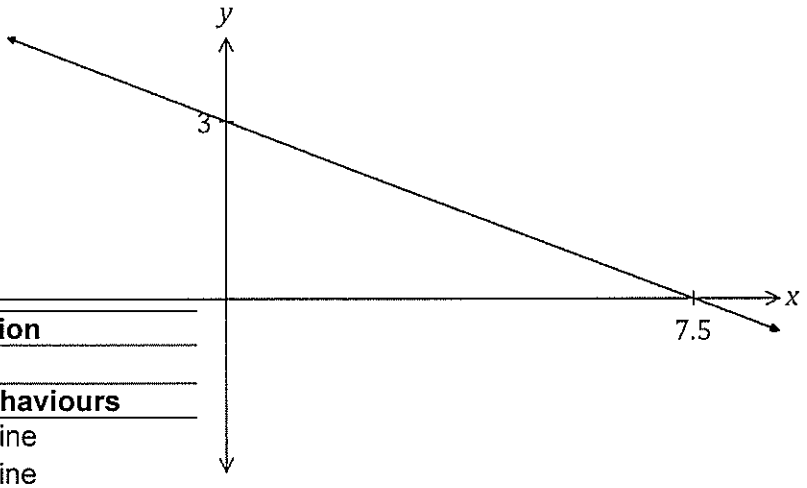
This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 10 **(7 marks)**

The variables x and y are related by the equation $2x + 5y = 15$.

(a) Sketch the graph of this relationship. (2 marks)



Solution
See graph
Specific behaviours
✓ y-intercept of line
✓ x-intercept of line

(b) Express y in terms of x and briefly explain why y is a function of x. (2 marks)

Solution
$y = 3 - \frac{2}{5}x$ For each x value there is just one y value, so one-to-one relationship.
Specific behaviours
✓ correct rule ✓ indicates use of vertical line test or 'one-to-one' relationship

(c) The domain of x is restricted to $-5 \leq x < 10$. State the range of y. (3 marks)

Solution
$x = -5, y = 5, \quad x = 10, y = -1$ $-1 < y \leq 5$
Specific behaviours
✓ correct lower value ✓ correct upper value ✓ correct inequalities

See next page

Question 11

(7 marks)

(a) The extension, E, of a spring is directly proportional to the lead weight , w, hung on the end of it. The extension was 15 mm when the weight hung on the spring was 120 g.

Write an equation relating the variables, E and w. (2 marks)

Solution
$E=kw$, $15 = k(120)$ $k = \frac{1}{8}$ $E = \frac{1}{8}w$
Specific behaviours
✓ solves for k ✓ writes equation

(b) Susan's pool can be emptied in 40 minutes using a small pump and in 15 minutes using a large pump. The pumps do not affect each other when used together. Determine, to the nearest minute, the time taken to empty the pool when both pumps are used. (5 marks)

Solution
$T = \frac{V}{R}$ where T= time, V = volume of tank, R = rate emptied $R_1 = \frac{V}{40}$, $R_2 = \frac{V}{15}$ $T = \frac{V}{R1+R2} = V \div (\frac{V}{40} + \frac{V}{15})$ $T = 1 \div (\frac{1}{40} + \frac{1}{15})$ $= 1 \div \frac{11}{120}$ $= \frac{120}{11}$ $= 10.91$ approx 11 mins
Specific behaviours
✓ identifies inverse proportion ✓ determines rate pumps empty in terms of V ✓ substitute rates into equation for T ✓ solves for time ✓ to nearest min. *NB choosing an arbitrary volume such as 40 is acceptable and simplifies working*

See next page

Question 12 **(9 marks)**

(a) Calculate the area of the minor segment that subtends an arc of 150° in a circle of diameter 190 cm. (2 marks)

Solution
$150° = \frac{5\pi}{6}, \quad r = \frac{190}{2} = 95$ $A = \frac{1}{2}(95)^2\left(\frac{5\pi}{6} - \sin\frac{5\pi}{6}\right) \approx 9\,557 \text{ cm}^2$
Specific behaviours
✓ converts angle, uses correct radius
✓ calculates area

(b) A chord of length 33 cm subtends an angle of $\frac{\pi}{11}$ at the centre of a circle. Calculate the radius of the circle. (2 marks)

Solution
$33 = 2r\sin\left(\frac{1}{2} \times \frac{\pi}{11}\right)$ $r \approx 116 \text{ cm}$
Specific behaviours
✓ substitutes into formula
✓ calculates radius

(c) Parallelogram $PQRS$ has side $PQ = 35$ cm, side $QR = 18$ cm and an area of 200 cm^2. Determine the lengths of the diagonals of $PQRS$. (5 marks)

Solution
$\frac{1}{2}(35)(18)\sin x = \frac{200}{2}$ $x = 18.51°, 161.49°$ $L_1 = \sqrt{35^2 + 18^2 - 2(35)(18)\cos 18.51}$ $\approx 18.8 \text{ cm}$ $L_2 = \sqrt{35^2 + 18^2 - 2(35)(18)\cos 161.49}$ $\approx 52.4 \text{ cm}$
Specific behaviours
✓ equation for half area
✓one angle of parallelogram
✓ second angle of parallelogram
✓length of one diagonal
✓ second correct length

See next page

Question 13 **(9 marks)**

Two points P and R have coordinates (3, 4) and (7,8) respectively.

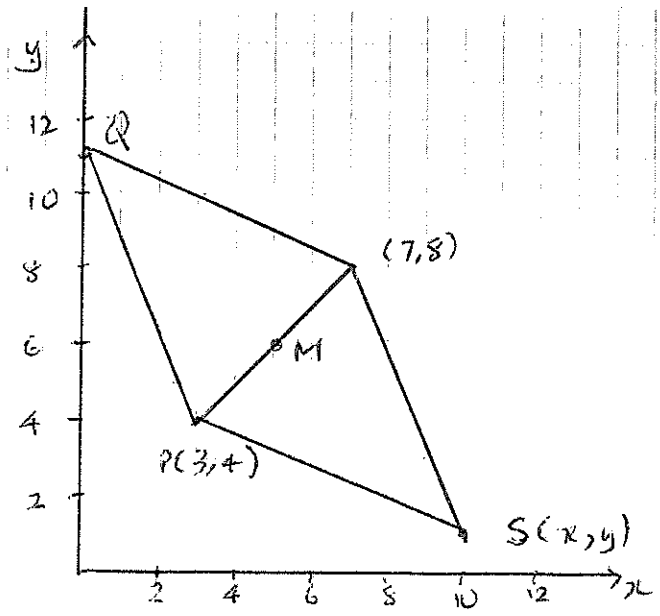
(a) Find the equation of the perpendicular bisector of PR. (4 marks)

Solution
Midpoint of $PR = \left(\frac{7+3}{2}, \frac{8+4}{2}\right) = (5, 6)$ $m_{PR} = \frac{8-4}{7-3} = 1$ m perpendicular = -1 eqtn of perpendicular bisector at (5, 6) : $6 = -1(5) + c$ $c = 11$ $\therefore$ eqtn is $y = -x + 11$
Specific behaviours
✓ correct midpoint ✓ correct gradient of PR and perpendicular ✓ calculates c ✓ correct equation

(b) $PQRS$ is a rhombus. Q is a point on the y axis and is equidistant from P and R.

Find

(i) the coordinates of Q (1 mark)



Solution
Q = (0, 11)
Specific behaviours
✓ correct coordinates f/t from (a)

See next page

(ii) the coordinates of *S*. Derive your answer mathematically.
(4 marks)

Solution
$PQ = \sqrt{((0-3)^2 + (11-4)^2)} = \sqrt{9+49} = \sqrt{58}$ Let S = (*x*,*y*) *PQ= PS = RS* $PS = \sqrt{(x-3)^2 + (y-4)^2} = \sqrt{58}$ $RS = \sqrt{(x-7)^2 + (y-8)^2} = \sqrt{58}$ S = (10, 1)
Specific behaviours
✓ calculates *PQ* ✓ eqtn for *PS* ✓ eqtn for *RS* ✓ correct coordinates for *S*

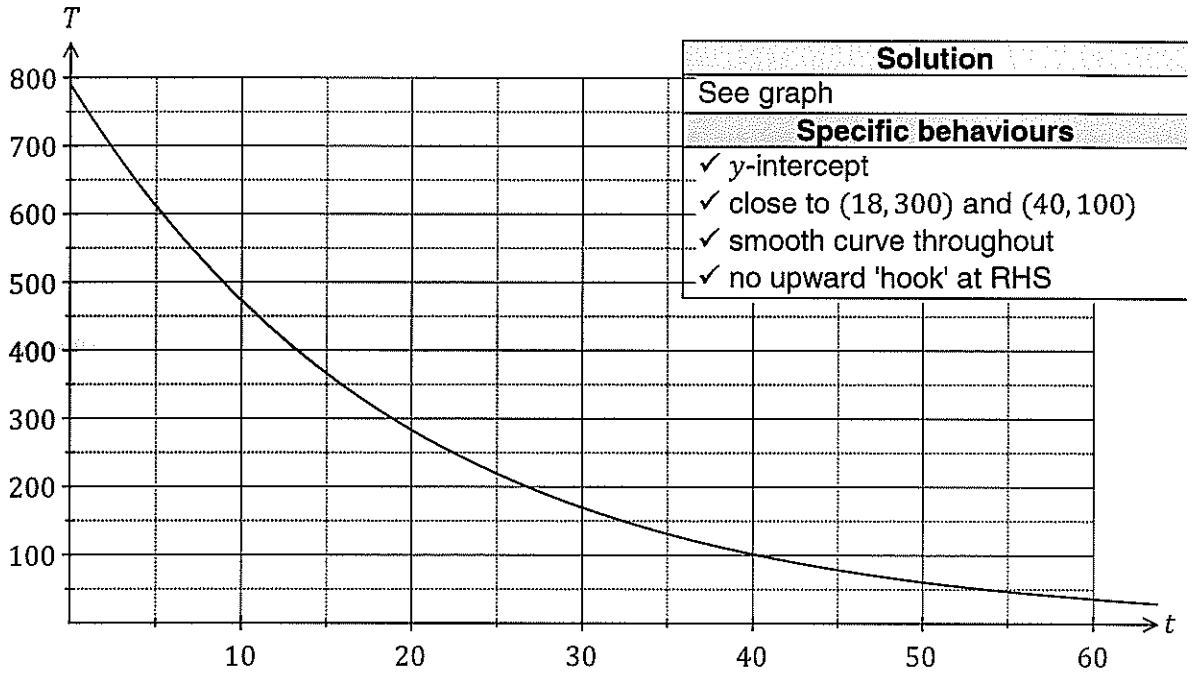
Question 14 **(9 marks)**

The temperature T of a cast taken out of an oven cools according to the model $T = 790(0.95)^t$, where t is the time in minutes since the cast was removed from the oven. T is measured in $°C$.

(a) Determine the fall in temperature of the cast during the first 6 minutes. (2 marks)

Solution
$T = 790(0.95)^6 \approx 581°C$ $\Delta T = 790 - 581 = 209°C$
Specific behaviours
✓ value of T when $t = 6$
✓ correct drop

(b) Graph the temperature of the cast against time on the axes below. (4 marks)



Solution
See graph
Specific behaviours
✓ y-intercept
✓ close to $(18, 300)$ and $(40, 100)$
✓ smooth curve throughout
✓ no upward 'hook' at RHS

(c) State the name of this type of function. (1 mark)

Solution
Exponential.
Specific behaviours
✓ correct name

(d) The temperature of the cast falls to room temperature of $20°C$.

(i) Determine the time taken for the cast to reach room temperature. (1 mark)

Solution
$790(0.95)^t = 20 \Rightarrow t = 71.7$ mins
Specific behaviours
✓ correct time

(ii) Comment on the usefulness of the model for large values of t. (1 mark)

Solution
For large values of t the model shows that $T \to 0$ but the temperature of the cast only falls to $20°C$ and so model not valid for large T.
Specific behaviours
✓ states not valid, with reason

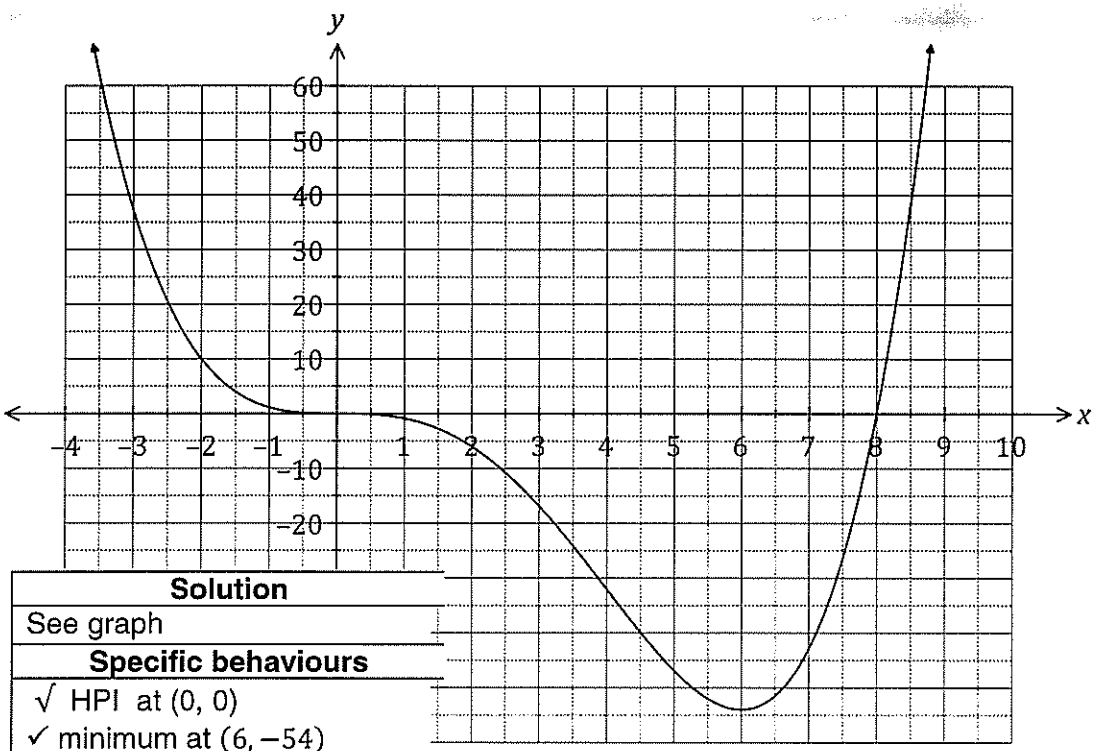
Question 15 **(7 marks)**

A function is defined by $f(x) = \frac{x^4}{8} - x^3$.

(a) Use the derivative $f'(x)$ to determine the coordinates of all stationary points of the function. (3 marks)

Solution
$f'(x) = \frac{1}{2}x^3 - 3x^2$
$\frac{1}{2}x^3 - 3x^2 = 0 \Rightarrow x = 0, x = 6$
$f(0) = 0, \quad f(6) = -54$
Stationary points at $(0, 0)$ and $(6, -54)$
Specific behaviours
✓ correct derivative
✓ correct zeros of derivative
✓ correct coordinates

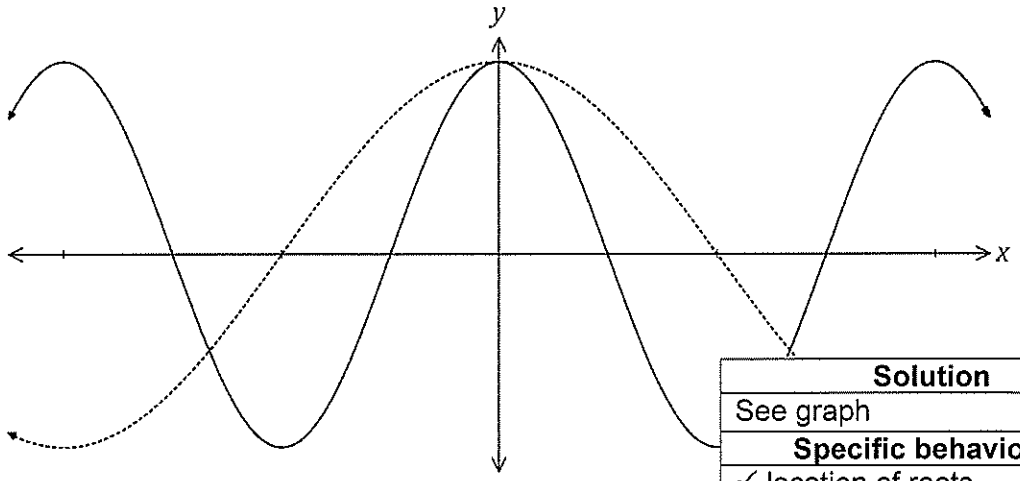
(b) Sketch the graph of $y = f(x)$ on the axes below. (4 marks)



Solution
See graph
Specific behaviours
√ HPI at (0, 0)
✓ minimum at $(6, -54)$
✓ root at $(8, 0)$
✓ correct curvature

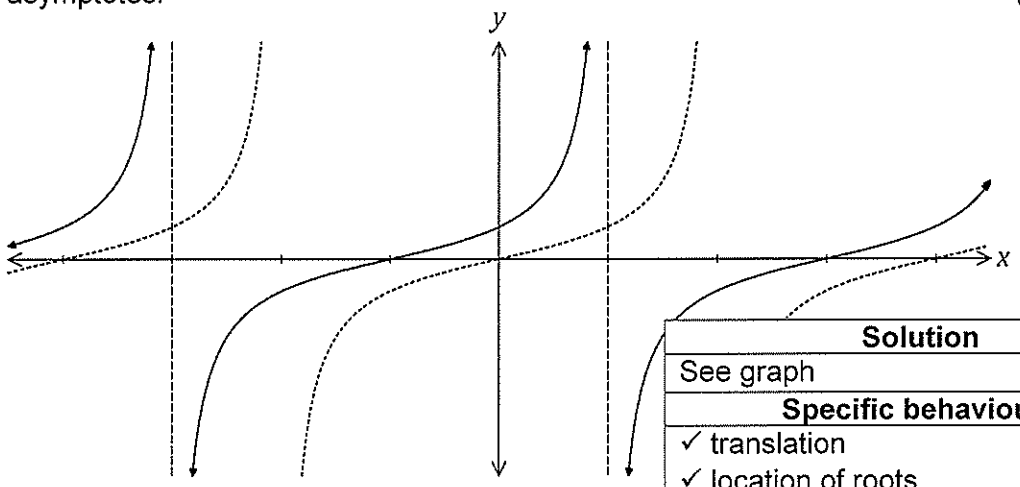
Question 16 **(7 marks)**

(a) The graph of $y = \cos x$ is shown below. On the same axes, sketch $y = \cos 2x$. (2 marks)



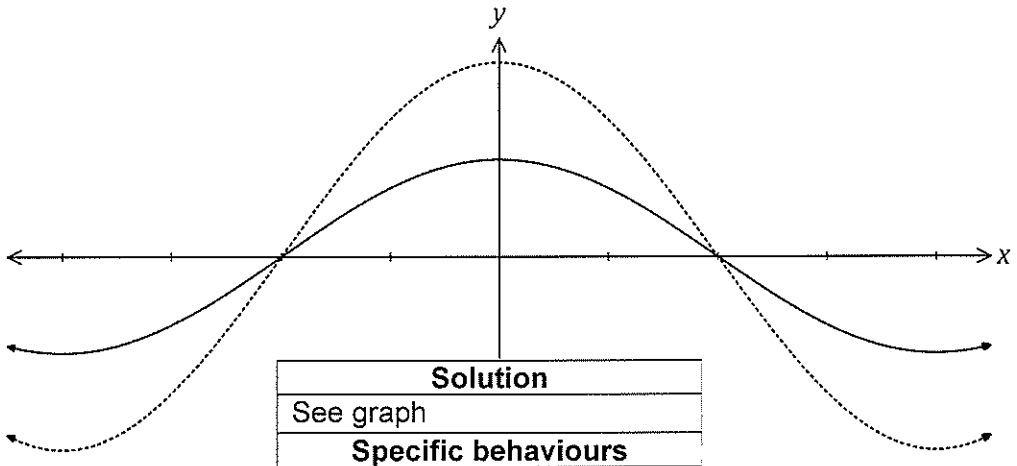
Solution
See graph
Specific behaviours
✓ location of roots
✓ correct amplitude/shape

(b) The graph of $y = \tan x$ is shown below. On the same axes, sketch $y = \tan\left(x + \frac{\pi}{4}\right)$, and all its asymptotes. (3 marks)



Solution
See graph
Specific behaviours
✓ translation
✓ location of roots
✓ asymptotes

(c) The graph of $y = 2\cos x$ is shown below. On the same axes, sketch $y = \sin\left(x + \frac{\pi}{2}\right)$. (2 marks)



Solution
See graph
Specific behaviours
✓ location of roots
✓ correct amplitude/shape

See next page

Question 17 **(8 marks)**

Two water containers, initially empty, are being filled with water. The amount of water added to **container** A each minute follows an arithmetic sequence, with 3 mL poured in during the first minute and 6 mL poured in during the second minute.

The amount of water added to **container** B each minute follows a geometric sequence, with 2 mL poured in during the first minute and 2.2 mL poured in during the second minute.

(a) The amount of water poured into **container** B during the n^{th} minute is given by $a(r)^{n-1}$. State the value of the constants a and r. (2 marks)

Solution
$a = 2$ $r = \frac{2.2}{2} = 1.1$
Specific behaviours
✓ value of a ✓ value of r

(b) Determine the total amount of water in **container** B at the end of the 25^{th} minute. (2 marks)

Solution
$S_{25} = \frac{2(1-1.1^{25})}{1-1.1}$ $= 197 \text{ mL}$
Specific behaviours
✓ uses sum formula ✓ correct amount

(c) How long does it take to fill **container** A with 360 mL of water? (2 marks)

Solution
$\frac{n}{2}(2(3) + (n-1)(3)) = 360$ $n = 15 \text{ minutes}$
Specific behaviours
✓ uses sum formula ✓ correct time

(d) **Container** B first holds more water than **container** A at the end of minute m.

(i) Determine the value of m. (1 mark)

Solution
$m = 59$
Specific behaviours
✓ correct value

(ii) State, to the nearest mL, how much more water B contains than A at this time. (1 mark)

Solution
$5516 - 5310 = 206 \text{ mL}$
Specific behaviours
✓ correct value

See next page

Question 18 **(8 marks)**

A council took a random sample of 155 properties from suburb A and 127 properties from suburb B. A total of 49 of the properties in the sample were in arrears with their rates, 27 of which were in suburb A. 'In arrears' means that payment of rates is overdue.

(a) Council officers wanted to choose 5 of the properties that were in arrears. How many different selections of properties are possible? (2 marks)

Solution
$\binom{49}{5} = 1\,906\,884$
Specific behaviours
✓ indicates use of nCr formula
✓ correct number

(b) Determine the probability that one randomly chosen property from the sample

(i) is in suburb B and is in arrears. (2 marks)

Solution
$P = \frac{49-27}{155+127} = \frac{22}{282}$ (≈ 0.078)
Specific behaviours
✓ numerator
✓ denominator

(ii) is not in arrears given that it is in suburb A. (1 mark)

Solution
$P = \frac{155-27}{155} = \frac{128}{155}$ (≈ 0.8258)
Specific behaviours
✓ correct probability

(c) Justifying your answer with conditional probabilities calculated to 2 decimal places, comment on whether being in arrears with rates is independent of the suburb the property is in. (3 marks)

Solution
$P(\text{Arrears}\|A) = \frac{27}{155} \approx 0.17$ $P(\text{Arrears}\|B) = \frac{22}{127} \approx 0.17$ Hence being in arrears is independent of suburb, as conditional probabilities are very similar.
Specific behaviours
✓ calculates $P(\text{Arrears}\|A)$
✓ calculates $P(\text{Arrears}\|B)$
✓ correct conclusion

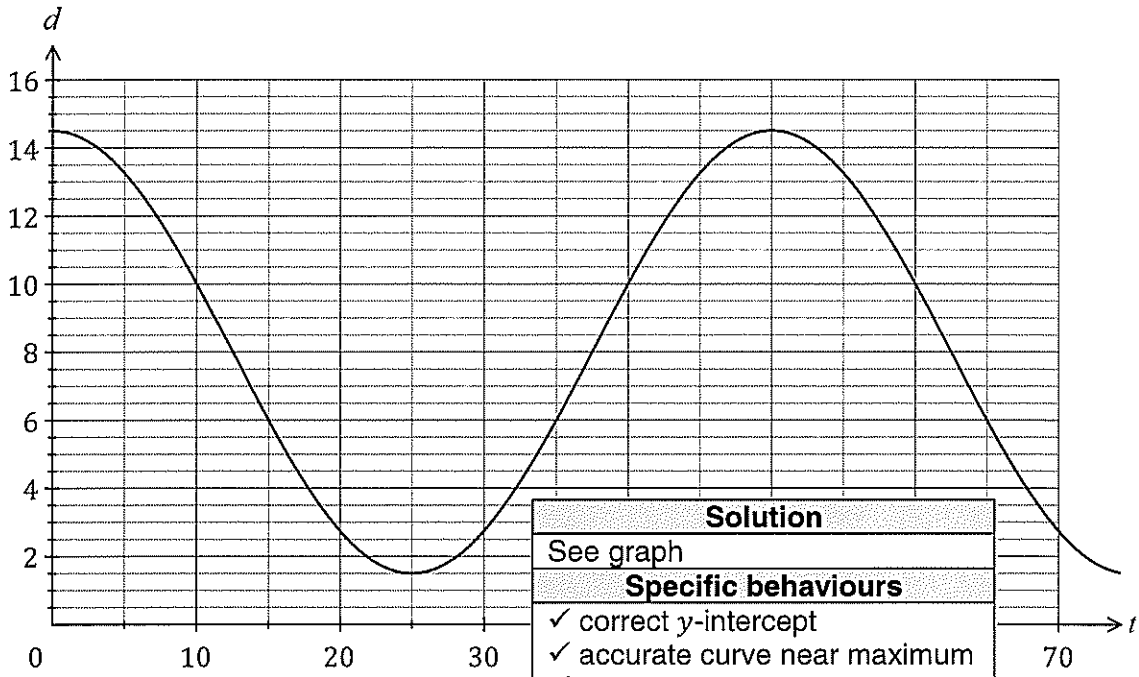
See next page

Question 19 **(8 marks)**

The height, h metres, above level ground of a seat on a steadily rotating Ferris wheel t seconds after observations began was given by

$$h = 6.5\cos\left(\frac{\pi t}{25}\right) + 8, \qquad t \geq 0.$$

(a) Draw the graph of the height of the seat against time on the axes below. (4 marks)



Solution
See graph
Specific behaviours
✓ correct y-intercept
✓ accurate curve near maximum
✓ accurate curve near minimum
✓ smooth curve throughout

(b) How long did the Ferris wheel take to complete one revolution? (1 mark)

Solution
50 seconds
Specific behaviours
✓ correct time

(c) At what time, when the seat was rising, did it first reach a height of 11 metres? (1 mark)

Solution
$t = 41.3$ s
Specific behaviours
✓ time that rounds to 41 s

(d) Determine the change in height of the seat between $t = 130$ and $t = 131$, giving your answer rounded to the nearest cm. (2 marks)

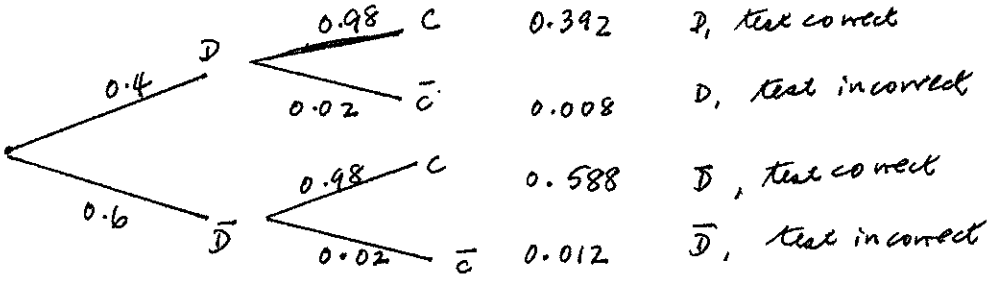
Solution
$h(130) = 2.74, \quad h(131) = 3.26$
$\delta h = 3.26 - 2.74 = 0.52$ m
Specific behaviours
✓ determines both heights
✓ states difference to nearest cm

See next page

Question 20 **(10 marks)**

It is known that 40% of all dogs in a country have a particular disease. A diagnostic test for the disease has been developed. The test gives a correct and true outcome 98% of the time. It can be assumed that the correct outcome of the test is independent of whether the dog has the disease.

(a) Construct an appropriate sample space to answer the questions below. (2 marks)



✓ 1st and 2nd branch
✓ outcomes

(b) A dog is randomly selected for the test from those in the country. Determine the probability that

(i) the dog has the disease, but the test indicates that it does not. (2 marks)

Solution
$P = 0.4 \times 0.02 = 0.008$
Specific behaviours
✓ indicates multiplication of correct probabilities
✓ correct probability

(ii) the dog actually has the disease if the test indicates that it does. (3 marks)

Solution
$\frac{0.392}{0.392 + 0.012}$
$= 0.9703$
Specific behaviours
✓ numerator
✓ denominator
✓ correct probability

(c) Two dogs are randomly selected for the test from those in the country. Determine the probability that just one of the dogs is diagnosed correctly. (3 marks)

Solution
P(1st correct and 2nd incorrect) or P(1st incorrect and 2nd correct)
= 2 [(0.392 + 0.588) x (0.008 + 0.012)]
= 2 (0.98 x 0.02)
= 2 (0.0196)
= 0.0392
Specific behaviours
✓ indicates correct method
✓ indicates prob. X 2
✓ correct probability

See next page

Question 21 **(9 marks)**

A pyramid with a rectangular base of length L and width w has perpendicular height h. The length of the base is five times its width and the sum of the width, length and height is 117 cm.

(a) Calculate the length, height and volume of the pyramid when $w = 15$ cm. (2 marks)

Solution
$L = 5 \times 15 = 75, \quad h = 117 - 15 - 75 = 27$ $V = \frac{1}{3}(15 \times 75) \times 27 = 10\,125 \text{ cm}^3$
Specific behaviours
✓ correct length and height ✓ correct volume

(b) Show that the volume of the pyramid is given by $V = 195w^2 - 10w^3$. (2 marks)

Solution
$L = 5w, \quad h = 117 - w - 5w = 117 - 6w$ $V = \frac{1}{3}(w \times 5w)(117 - 6w)$ $= 195w^2 - 10w^3$
Specific behaviours
✓ expressions for length and height ✓ substitutes width, length and height correctly

(c) Use calculus to determine the maximum volume of the pyramid and state the dimensions required to achieve this. (5 marks)

Solution
$\frac{dV}{dw} = 390w - 30w^2$ $390w - 30w^2 = 0 \Rightarrow w = 0, 13$ $V_{max} = 195(13)^2 - 10(13)^3 = 10\,985 \text{ cm}^3$ $w = 13 \text{ cm}, \quad L = 65 \text{ cm}, \quad h = 39 \text{ cm}$
Specific behaviours
✓ correct derivative using given variables ✓ solves derivative equal to zero √ proves max, sign test ✓ correct maximum volume ✓ correct dimensions

End of questions